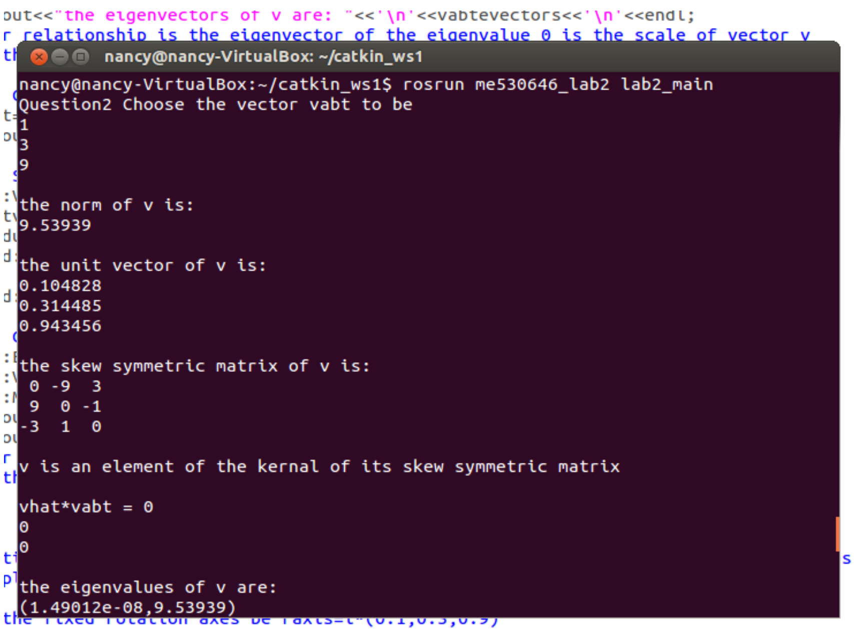
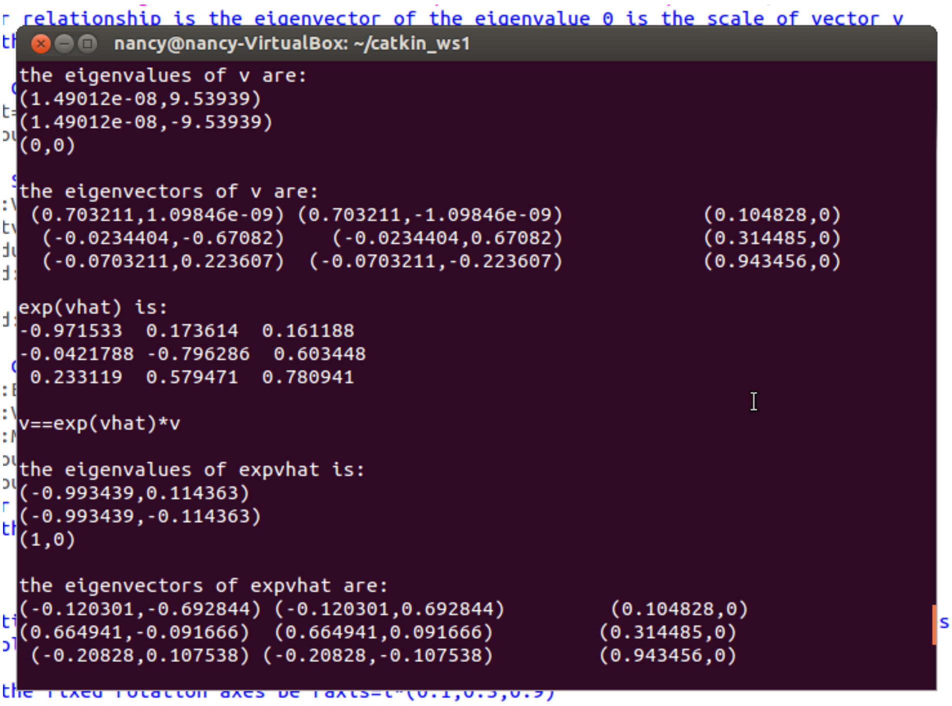
Lab2 Answers

The experiments for the calculating and demonstrations are written in the lab2_main.cpp, each question corresponding to a block of codes with comments as the specified question. You can get nearly all the answers by running the lab2_main.cpp. In the animation in Rviz, the frame rotations show first as of Question3, and then the frames of transformations show as of Question 5, however, because of the norm of the parameters differ largely, the Rviz Display should be adjusted a little bit to see the results of Question 5.

1. Question 2

   Have picked the vector v=$\begin{bmatrix}1\\3\\9\end{bmatrix}$, here is the screenshots for this question.





The answers are as below.

(a) ||v||= 9.53939

(b) $\bar{v}$=v*(1/ ||v||)= $\begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$

(c) $\hat{v}$= $\begin{bmatrix} 0 & -9 & 3 \\ 9 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$

(d) has shown $\hat{v} * v = 0$, so v$\in \mathrm{Ker}(\hat{v})$

(e) The eigenvalues of v are: 9.53939i, -9.53939i, 0

The eigenvectors of v are:

$$\begin{bmatrix} 0.703 \\ -0.0234 - 0.67i \\ -0.0703 + 0.224i \end{bmatrix}, \begin{bmatrix} 0.703 \\ -0.0234 + 0.67i \\ -0.0703 - 0.224i \end{bmatrix}, \begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$$

Their relation is: the eigenvector to the eigenvalue of 0 is the unit vector of v, which also means $\hat{v} * v = 0$, and the other two eigenvectors are conjugate.

(f) $e^{\hat{v}}$= $\begin{bmatrix} -0.972 & 0.174 & 0.161 \\ -0.042 & -0.796 & 0.603 \\ 0.233 & 0.579 & 0.781 \end{bmatrix}$

(g) v =$e^{\hat{v}}$* v has been shown

(h) The eigenvalues of $e^{\hat{v}}$ are: -0.993+0.114i, -0.993-0.114i, 1

The eigenvectors of $e^{\hat{v}}$ are:

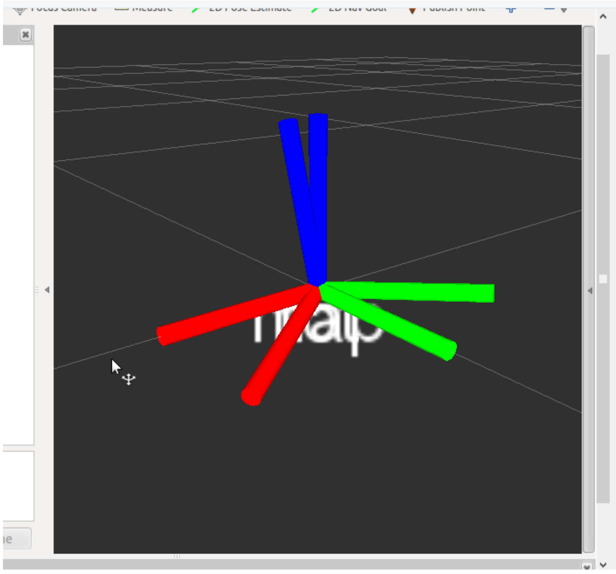
$\begin{bmatrix} -0.12-0.693\mathrm{i} \\ 0.665-0.092\mathrm{i} \\ -0.208+0.108\mathrm{i} \end{bmatrix}$, $\begin{bmatrix} -0.12+0.693\mathrm{i} \\ 0.665+0.092\mathrm{i} \\ -0.208-0.108\mathrm{i} \end{bmatrix}$, $\begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$

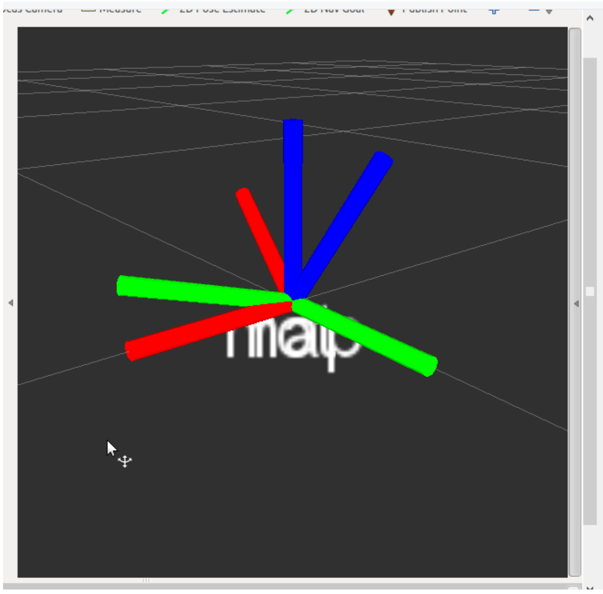
Their relation is: the eigenvector to the eigenvalue of 1 is the unit vector of v, which also means 1*v =$e^{\hat{v}}$* v, and the other two eigenvectors are conjugate.

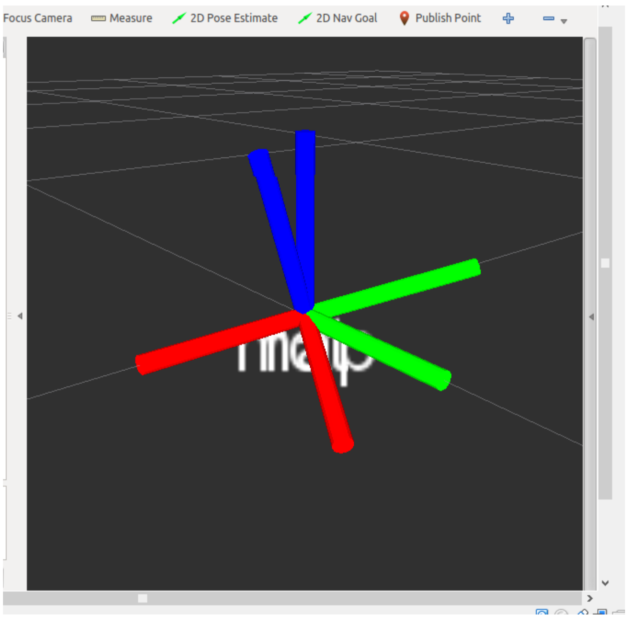
2. Question 3

Rotation matrices have been generated with axis$(0.1\ 0.3\ 0.9)^{\mathrm{T}}$, and different angles from around 0.095 to 28 radius.

Simply run lab2_main.cpp, you can see the animation. Some screenshots of the Rviz animation are as below.

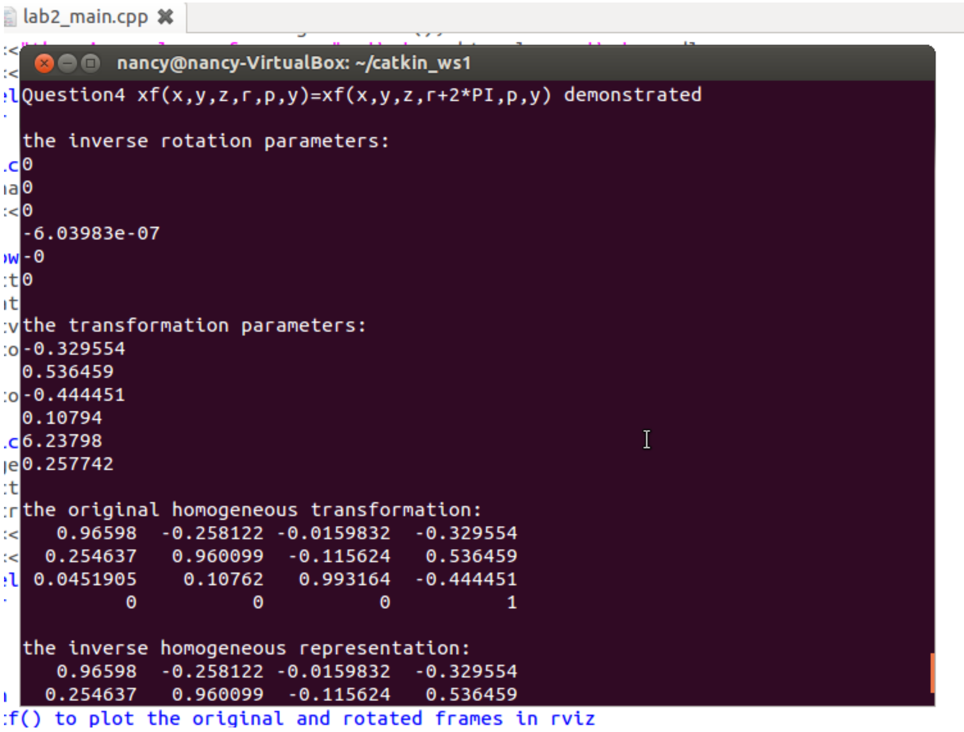


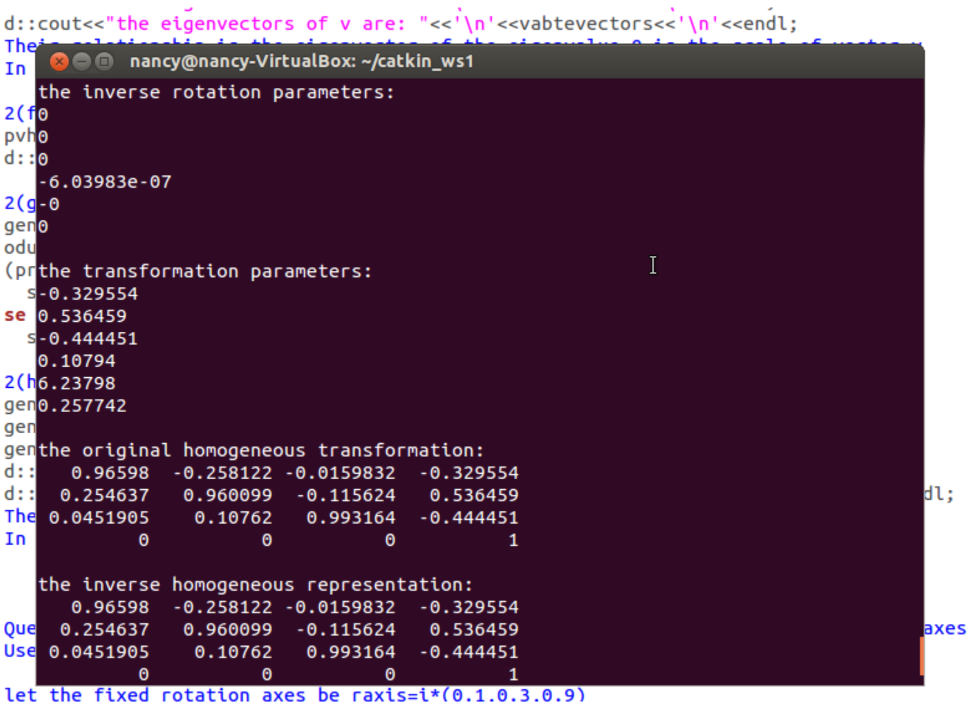



Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point

3. Question 4

Some screenshots of the results of the calculating are as below.





(a) The eigen::Vector6f::Random() function has been used to get (x, y, z, r, p, y), and has demonstrated xf(x, y, z, r, p, y)=xf(x, y, z, r + 2 π, p, y), ∀ x, y, z, r, p, y ∈ R

(b) For some homogeneous matrices, for example a homoTransf matrix, if homoTransf(0, 0)=homoTransf(1, 0)=0, then by applying xfinv(homoTransf) only x, y, z, p, and the relationship of angles r and y can be got, while neither r or y can be got. Besides, when homoTranf= xf(x, y, z, r, p+2 π, y), xfinv(homoTransf)= (x, y, z, r, p, y)!= (x, y, z, r, p+2 π, y).

(c) It has been shown that (0,0,0,4π,0,0)!= xfinv(xf(0,0,0,4π,0,0))= (0,0,0,0,0,0), and can be seen in the screenshot.

(d) It has been shown that for arbitrary (x, y, z, r, p, y), xfinv() for whose homotransf is well-defined, homotransf = xf(xfinv(homotransf), and it can be seen in the screenshot above.

In the example, homotransf = xf(xfinv(homotransf)

$$= \begin{bmatrix} 0.966 & -0.258 & -0.016 & -0.33 \\ 0.255 & 0.96 & -0.116 & 0.536 \\ 0.045 & 0.108 & 0.993 & 0.444 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4. Question 5

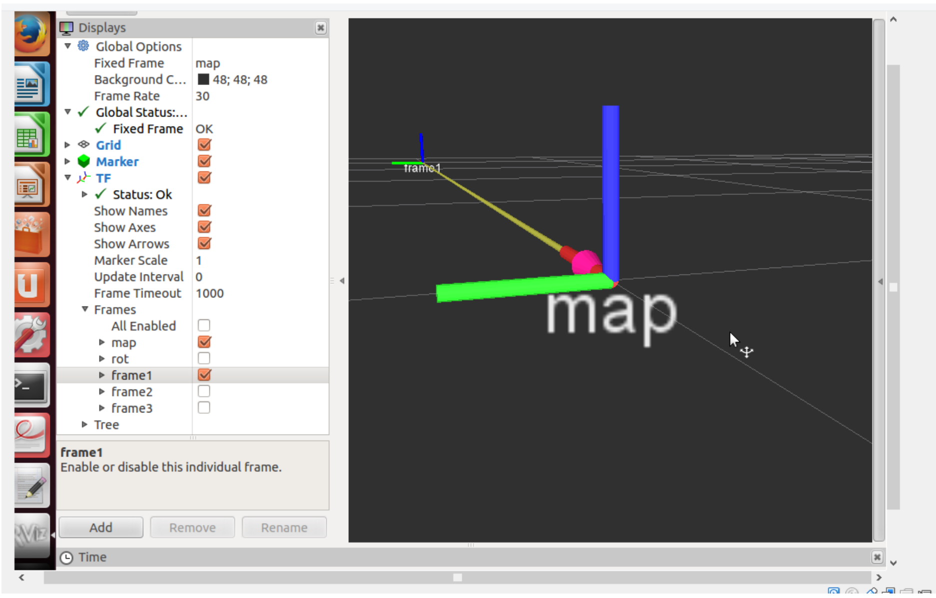
(a) $$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & q_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(q_2) & -\sin(q_2) & 0 \\ 0 & \sin(q_2) & \cos(q_2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

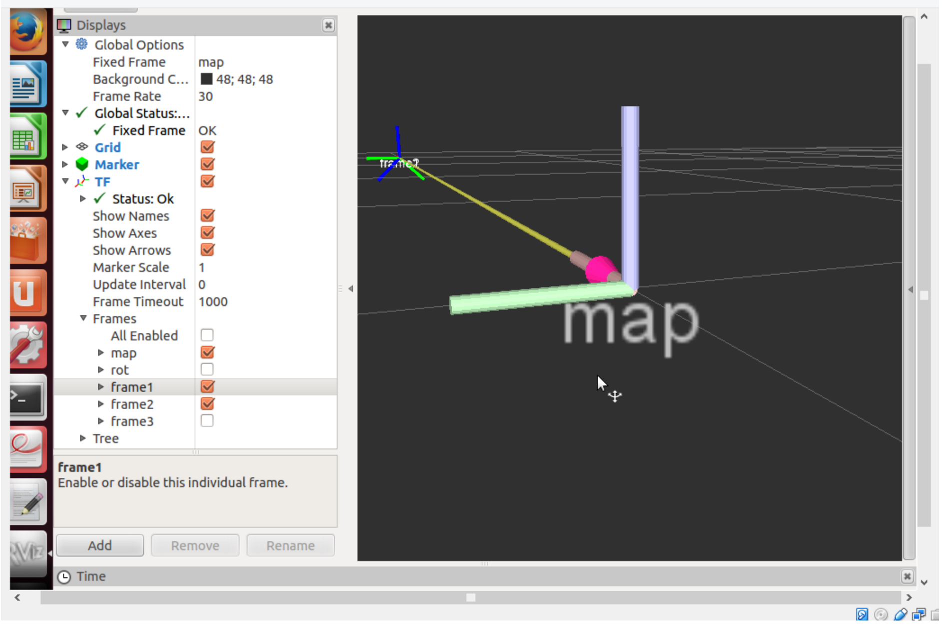
$$H_3^0 = H_1^0 * H_2^1 * H_3^2 = \begin{bmatrix} 1 & 0 & 0 & q_1 \\ 0 & \cos(q_2) & -\sin(q_2) & -q_3 * \sin(q_2) \\ 0 & \sin(q_2) & \cos(q_2) & q_3 * \cos(q_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $q_1 = q_2 = q_3 = \frac{5\pi}{4}$ have been chosen, and has chosen 5 points in Frame3 to get 3 vectors plotted in Frame3.

plot of Frame1 with transformation H_1^0 in relation to the world frame:

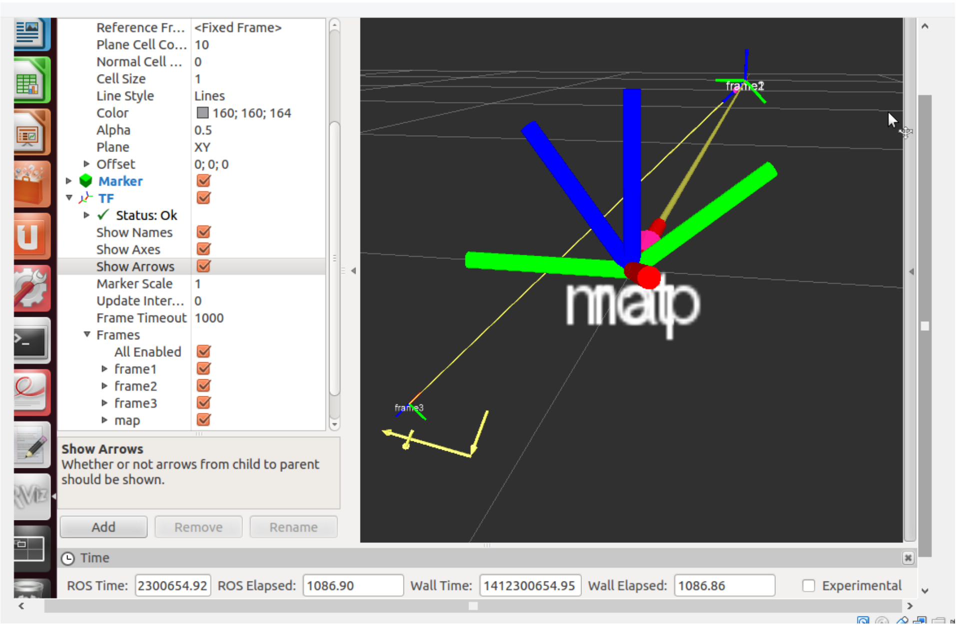


plot of Frame2 with transformation H_2^1 in relation to Frame1:



plot of Frame3 with transformation H_3^2 in relation to Frame2,

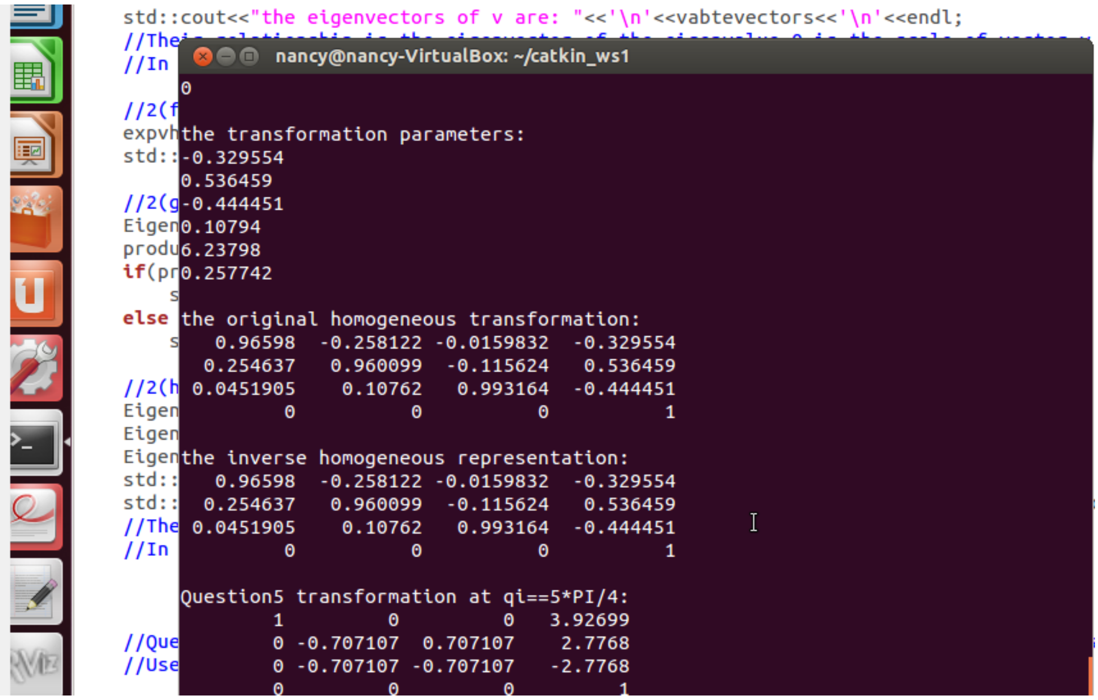
and the 3 vectors with respect to Frame3:



(c) When $q_1 = q_2 = q_3 = \frac{5\pi}{4}$, $H_3^0 = \begin{bmatrix} 1 & 0 & 0 & \frac{5\pi}{4} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{5\sqrt{2}\pi}{8} \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{5\sqrt{2}\pi}{8} \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

and using the getTransformation(), get the transformation

$H_3^0 = \begin{bmatrix} 1 & 0 & 0 & 3.927 \\ 0 & -0.707 & 0.707 & 2.777 \\ 0 & -0.707 & -0.707 & -2.777 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, as shown in the screenshot below.



They are equal.