

Lab2 Answers

The experiments for the calculating and demonstrations are written in the lab2_main.cpp, each question corresponding to a block of codes with comments as the specified question. You can get nearly all the answers by running the lab2_main.cpp. In the animation in Rviz, the frame rotations show first as of Question3, and then the frames of transformations show as of Question 5, however, because of the norm of the parameters differ largely, the Rviz Display should be adjusted a little bit to see the results of Question 5.

1. Question 2

Have picked the vector $v = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$, here is the screenshots for this

```
out<<"the eigenvectors of v are: "<<'\\n'<<vabteveectors<<'\\n'<<endl;
r relationship is the eigenvector of the eigenvalue 0 is the scale of vector v
ti
nancy@nancy-VirtualBox: ~/catkin_ws1
nancy@nancy-VirtualBox:~/catkin_ws1$ rosrn me530646_lab2 lab2_main
Question2 Choose the vector vabt to be
ti
pi
3
9
ti
the norm of v is:
ti
9.53939
di
the unit vector of v is:
ti
0.104828
di
0.314485
ti
0.943456
ti
the skew symmetric matrix of v is:
ti
0 -9 3
ti
9 0 -1
ti
-3 1 0
ti
r v is an element of the kernal of its skew symmetric matrix
ti
vhat*vabt = 0
ti
0
ti
0
ti
the eigenvalues of v are:
ti
(1.49012e-08,9.53939)
the fixed rotation axes be faxis=c(0.1,0.3,0.9)
```

question.

```

relationship is the eigenvector of the eigenvalue 0 is the scale of vector v
nancy@nancy-VirtualBox: ~/catkin_ws1
the eigenvalues of v are:
(1.49012e-08,9.53939)
(1.49012e-08,-9.53939)
(0,0)

the eigenvectors of v are:
(0.703211,1.09846e-09) (0.703211,-1.09846e-09) (0.104828,0)
(-0.0234404,-0.67082) (-0.0234404,0.67082) (0.314485,0)
(-0.0703211,0.223607) (-0.0703211,-0.223607) (0.943456,0)

exp(vhat) is:
-0.971533 0.173614 0.161188
-0.0421788 -0.796286 0.603448
0.233119 0.579471 0.780941

v==exp(vhat)*v

the eigenvalues of expvhat is:
(-0.993439,0.114363)
(-0.993439,-0.114363)
(1,0)

the eigenvectors of expvhat are:
(-0.120301,-0.692844) (-0.120301,0.692844) (0.104828,0)
(0.664941,-0.091666) (0.664941,0.091666) (0.314485,0)
(-0.20828,0.107538) (-0.20828,-0.107538) (0.943456,0)

```

The answers are as below.

(a) $||v|| = 9.53939$

(b) $\bar{v} = v * (1 / ||v||) = \begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$

(c) $\hat{v} = \begin{bmatrix} 0 & -9 & 3 \\ 9 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$

(d) has shown $\hat{v} * v = 0$, so $v \in \text{Ker}(\hat{v})$

(e) The eigenvalues of v are: $9.53939i, -9.53939i, 0$

The eigenvectors of v are:

$$\begin{bmatrix} 0.703 \\ -0.0234 - 0.67i \\ -0.0703 + 0.224i \end{bmatrix}, \begin{bmatrix} 0.703 \\ -0.0234 + 0.67i \\ -0.0703 - 0.224i \end{bmatrix}, \begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$$

Their relation is: the eigenvector to the eigenvalue of 0 is the unit vector of v, which also means $\hat{v} * v = 0$, and the other two eigenvectors are conjugate.

$$(f) \quad e^{\hat{v}} = \begin{bmatrix} -0.972 & 0.174 & 0.161 \\ -0.042 & -0.796 & 0.603 \\ 0.233 & 0.579 & 0.781 \end{bmatrix}$$

(g) $v = e^{\hat{v}} * v$ has been shown

(h) The eigenvalues of $e^{\hat{v}}$ are: $-0.993+0.114i$, $-0.993-0.114i$, 1

The eigenvectors of $e^{\hat{v}}$ are:

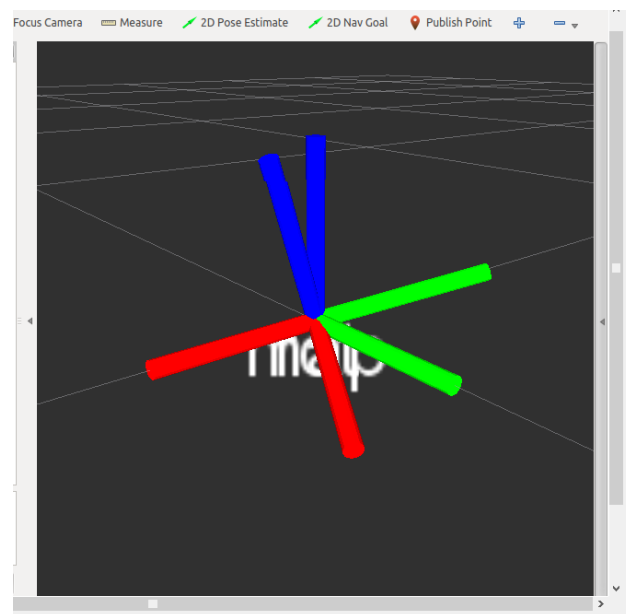
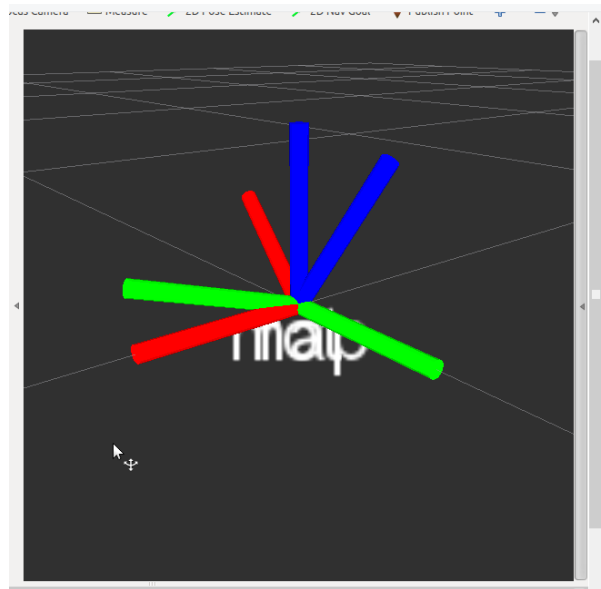
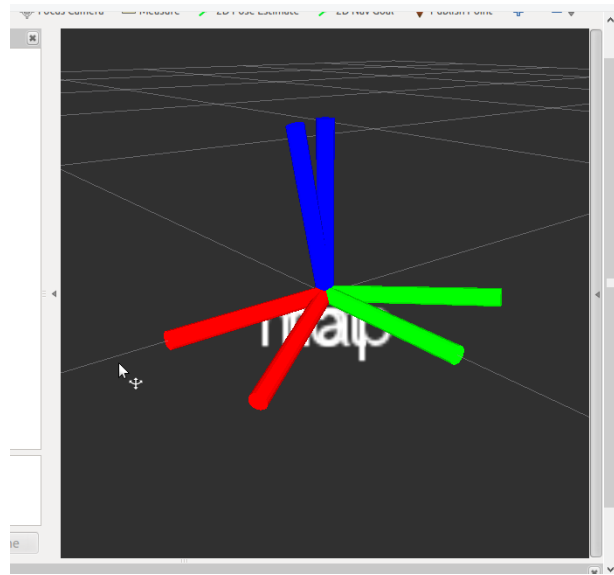
$$\begin{bmatrix} -0.12 - 0.693i \\ 0.665 - 0.092i \\ -0.208 + 0.108i \end{bmatrix}, \begin{bmatrix} -0.12 + 0.693i \\ 0.665 + 0.092i \\ -0.208 - 0.108i \end{bmatrix}, \begin{bmatrix} 0.104828 \\ 0.314485 \\ 0.943456 \end{bmatrix}$$

Their relation is: the eigenvector to the eigenvalue of 1 is the unit vector of v , which also means $1 * v = e^{\hat{v}} * v$, and the other two eigenvectors are conjugate.

2. Question 3

Rotation matrices have been generated with axis $(0.1 \ 0.3 \ 0.9)^T$, and different angles from around 0.095 to 28 radius.

Simply run `lab2_main.cpp`, you can see the animation. Some screenshots of the Rviz animation are as below.



3. Question 4

Some screenshots of the results of the calculating are as below.

```
lab2_main.cpp x
nancy@nancy-VirtualBox: ~/catkin_ws1
Question4 xf(x,y,z,r,p,y)=xf(x,y,z,r+2*PI,p,y) demonstrated
the inverse rotation parameters:
-6.03983e-07
the transformation parameters:
-0.329554
0.536459
-0.444451
0.10794
6.23798
0.257742
the original homogeneous transformation:
0.96598 -0.258122 -0.0159832 -0.329554
0.254637 0.960099 -0.115624 0.536459
0.0451905 0.10762 0.993164 -0.444451
0 0 0 1
the inverse homogeneous representation:
0.96598 -0.258122 -0.0159832 -0.329554
0.254637 0.960099 -0.115624 0.536459
f() to plot the original and rotated frames in rviz
d::cout<<"the eigenvectors of v are: "<<'\\n'<<vabteveectors<<'\\n'<<endl;
The relationship in the eigenvectors of the singular 0 is the only of system
In the inverse rotation parameters:
-6.03983e-07
the transformation parameters:
-0.329554
0.536459
-0.444451
0.10794
6.23798
0.257742
the original homogeneous transformation:
0.96598 -0.258122 -0.0159832 -0.329554
0.254637 0.960099 -0.115624 0.536459
0.0451905 0.10762 0.993164 -0.444451
0 0 0 1
the inverse homogeneous representation:
0.96598 -0.258122 -0.0159832 -0.329554
0.254637 0.960099 -0.115624 0.536459
0.0451905 0.10762 0.993164 -0.444451
0 0 0 1
let the fixed rotation axes be raxis=i*(0.1.0.3.0.9)
```

- (a) The `eigen::Vector6f::Random()` function has been used to get (x, y, z, r, p, y) , and has demonstrated $xf(x, y, z, r, p, y) = xf(x, y, z, r + 2\pi, p, y)$, $\forall x, y, z, r, p, y \in \mathbb{R}$
- (b) For some homogeneous matrices, for example a `homoTransf` matrix, if `homoTransf(0, 0) = homoTransf(1, 0) = 0`, then by applying `xfinv(homoTransf)` only x, y, z, p , and the relationship of angles r and y can be got, while neither r or y can be got. Besides, when `homoTranf = xf(x, y, z, r, p+2 π , y)`, `xfinv(homoTransf) = (x, y, z, r, p, y) != (x, y, z, r, p+2 π , y)`.
- (c) It has been shown that $(0,0,0,4\pi,0,0) != \text{xfinv}(xf(0,0,0,4\pi,0,0)) = (0,0,0,0,0,0)$, and can be seen in the screenshot.
- (d) It has been shown that for arbitrary (x, y, z, r, p, y) , `xfinv()` for whose `homotransf` is well-defined, `homotransf = xf(xfinv(homotransf))`, and it can be seen in the screenshot above.

In the example, `homotransf = xf(xfinv(homotransf))`

$$= \begin{bmatrix} 0.966 & -0.258 & -0.016 & -0.33 \\ 0.255 & 0.96 & -0.116 & 0.536 \\ 0.045 & 0.108 & 0.993 & 0.444 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4. Question 5

$$(a) \quad H_1^0 = \begin{bmatrix} 1 & 0 & 0 & q_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(q_2) & -\sin(q_2) & 0 \\ 0 & \sin(q_2) & \cos(q_2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

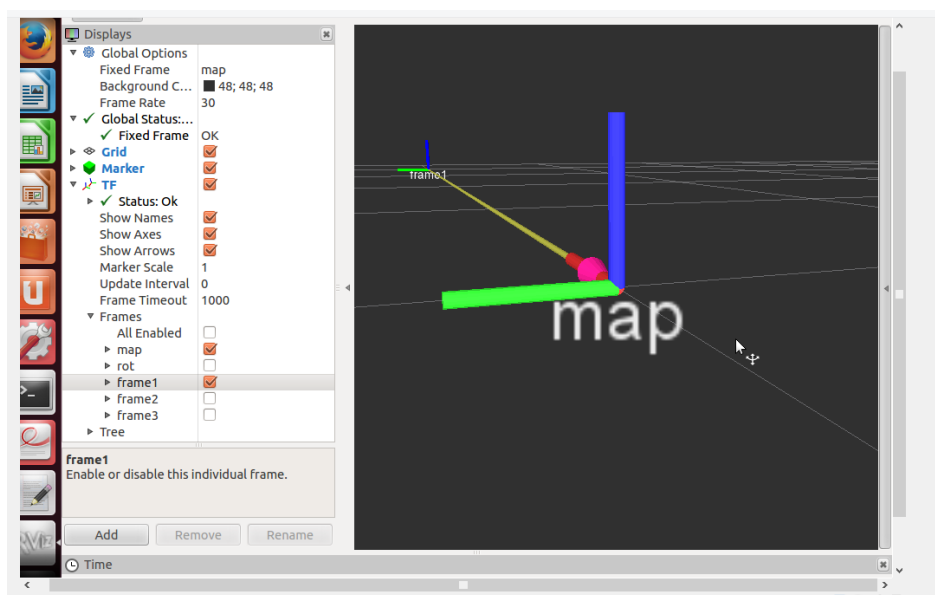
$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 * H_2^1 * H_3^2 = \begin{bmatrix} 1 & 0 & 0 & q_1 \\ 0 & \cos(q_2) & -\sin(q_2) & -q_3 * \sin(q_2) \\ 0 & \sin(q_2) & \cos(q_2) & q_3 * \cos(q_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

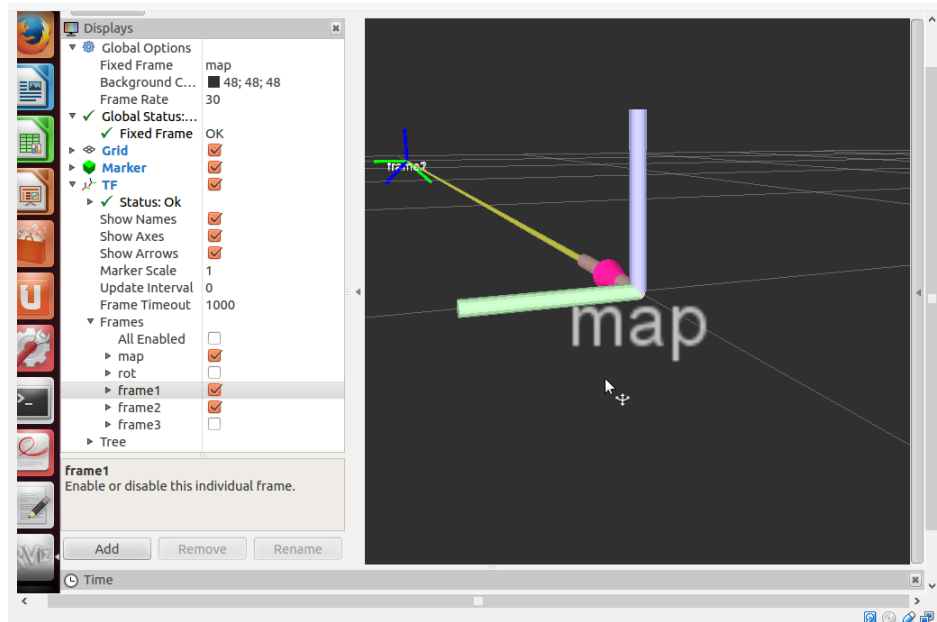
(b) $q_1 = q_2 = q_3 = \frac{5\pi}{4}$ have been chosen, and has chosen 5 points in Frame3 to get 3 vectors plotted in Frame3.

plot of Frame1 with transformation H_1^0 in relation to the world

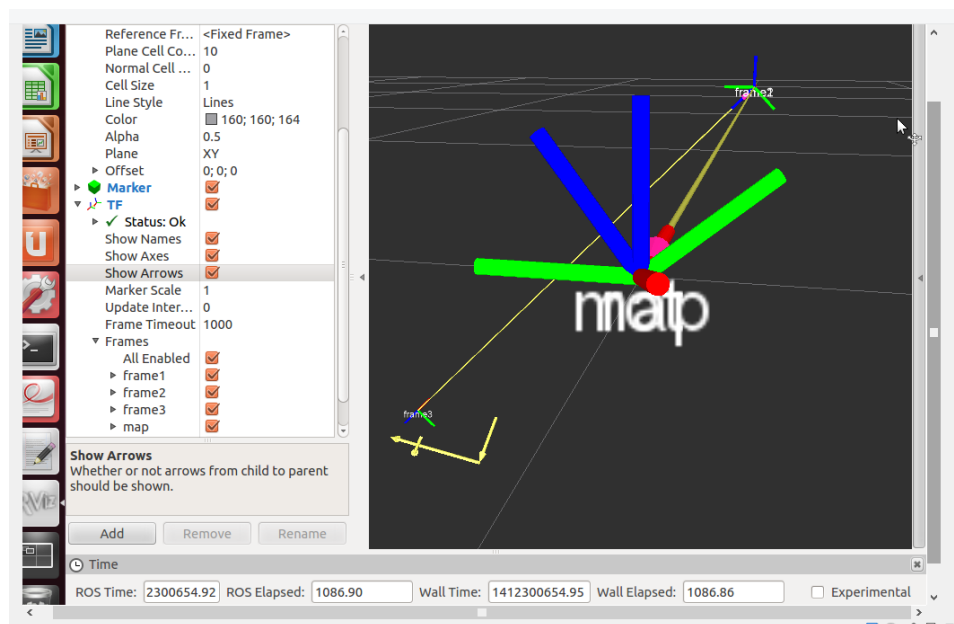
frame:



plot of Frame2 with transformation H_2^1 in relation to Frame1:



plot of Frame3 with transformation H_3^2 in relation to Frame2,
and the 3 vectors with respect to Frame3:

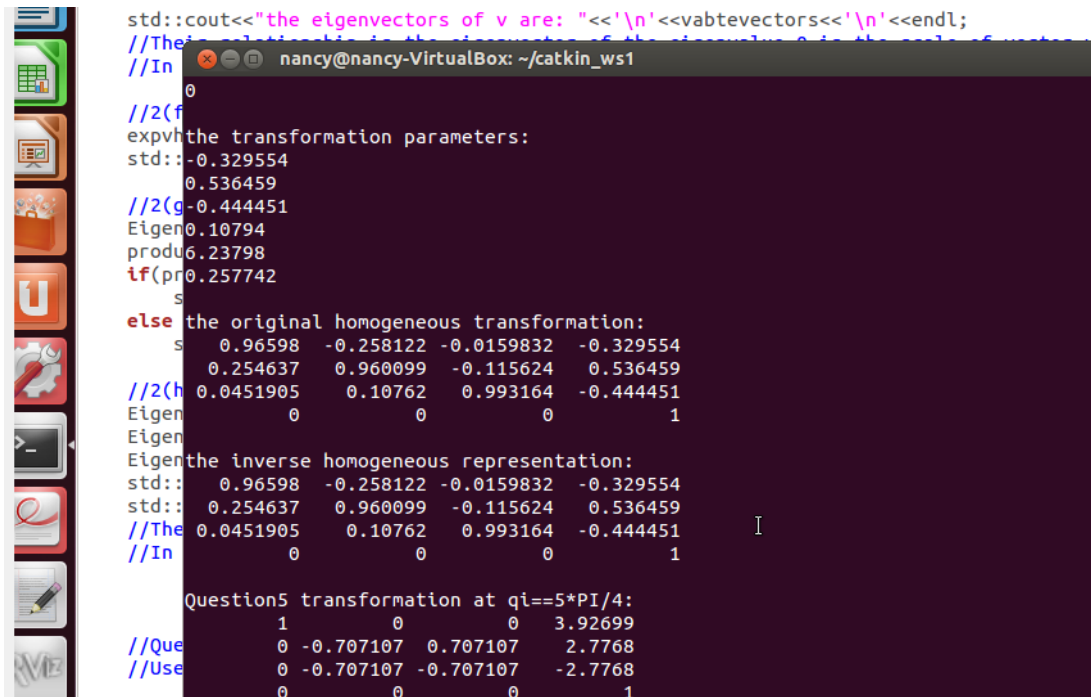


(c) When $q_1 = q_2 = q_3 = \frac{5\pi}{4}$, $H_3^0 = \begin{bmatrix} 1 & 0 & 0 & \frac{5\pi}{4} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{5\sqrt{2}\pi}{8} \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{5\sqrt{2}\pi}{8} \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

and using the `getTransformation()`, get the transformation

$$H_3^0 = \begin{bmatrix} 1 & 0 & 0 & 3.927 \\ 0 & -0.707 & 0.707 & 2.777 \\ 0 & -0.707 & -0.707 & -2.777 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ as shown in the}$$

screenshot below.



```
std::cout<<"the eigenvectors of v are: "<<"\n"<<vabteveectors<<"\n"<<endl;
//The relationship in the eigenvectors of the eigenvector 0 is the scale of vectors
//In
0
//2(f
expvthe transformation parameters:
std::-0.329554
0.536459
//2(g-0.444451
Eigen0.10794
produ6.23798
if(pr0.257742
s
else the original homogeneous transformation:
s 0.96598 -0.258122 -0.0159832 -0.329554
0.254637 0.960099 -0.115624 0.536459
//2(h 0.0451905 0.10762 0.993164 -0.444451
Eigen 0 0 0 1
Eigen
Eigen the inverse homogeneous representation:
std:: 0.96598 -0.258122 -0.0159832 -0.329554
std:: 0.254637 0.960099 -0.115624 0.536459
//The 0.0451905 0.10762 0.993164 -0.444451
//In 0 0 0 1

Question5 transformation at qi==5*PI/4:
1 0 0 3.92699
//Que 0 -0.707107 0.707107 2.7768
//Use 0 -0.707107 -0.707107 -2.7768
0 0 0 1
```

They are equal.